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Master QFin, CTFI
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Hints

- Please mark every sheet you upload with your name and Mat.Nr. and leave a margin for corrections
- Please take care of the time, late submissions are not accepted.
- By uploading your solution you implicitly state that you solved the assignment completely on your own.
- Good luck !

1. Feynman Kac. (4 points) Consider the solution X of the SDE $dX_t = \kappa(\theta - X_t)dt + \sigma\sqrt{X_t}dW_t$ for parameters $\kappa, \theta, \sigma > 0$. (It is known that a unique solution exists for every initial value $x > 0$ and that the solution is nonnegative for all t.) Define the function

$$F(t,x) = E_x\Big(\exp\big(-\int_0^{T-t} X_s ds\big)\Big), \quad (t,x) \in [0,T] \times (0,\infty)\,.$$

Use the Feynman Kac formula to derive a terminal value problem for F.

2 Properties of option prices in Black Scholes. (4 points) The price of an option in the Black Scholes model is independent of the drift μ of the asset. Why is this reasonable from an economic point of view? Is the independence of the drift a reasonable assumption also for a buy-and-hold investor? Give a *short* discussion of both points.

3. Black Scholes model and binary option. Consider in the context of the Black Scholes model with stock price dynamics $dS_t = \mu S_t dt + \sigma S_t dW_t$, initial stock price $S_0 > 0$ and with money market account $B_t = \exp(rt)$ for $r > 0$ a so-called binary option with payoff $h(S_T) = 1_{\{S_T \geq K\}}$ for some $K > 0$.

a) (4 points) Use the risk neutral pricing formula to show that the price of the option at time t equals

$$u(t,S) = N(d_2) \text{ where } d_2 = \frac{\frac{\ln S}{\ln K} + (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}},$$

and where N is the standard normal distribution function.

b (4 points) Construct a delta neutral hedging strategy for the option and compute explicitly the stock position. Discuss qualitatively potential problems in the implementation of the strategy. Hint: consider the value of the option delta for $S \approx K$ and a small time to maturity.